Electrical Engineering Dept., I.I.T. Delhi

Written Test for Ph.D. Admissions (Communication)

Time: 45 Minutes **Max. Marks: 13**

Write only the final answer after each question. No Overwriting.

Pages $1, 2, 3$ **contain questions.You can use page** 4 **for rough work.**

Each question carries 1 **mark.**

Name: **Application No:** **Signature:**

1) Find the value of

$$\lim_{x \to 0} \frac{xe^x - e^x + 1}{x \sin(x)}$$

Answer:

2) Find the eigenvalues of a real-valued 2×2 matrix whose diagonal entries are 1 and off-diagonal entries are ρ.

Answer:

3) If X is a real-valued random variable uniformly distributed in $(-1, 1)$ then what is the probability density function of the random variable $Y = X^3$?

Answer:

4) Consider two vectors $\vec{A} = 1\,\widehat{i} + 3\,\widehat{j}$ and $\vec{B} = 2\,\widehat{i} + 7\,\widehat{j}$. Find the dot product and cross product of these two vectors.

   Answer:

5) Let X and Y be independent real-valued random variables having probability density functions (pdf) $f_X(\cdot)$ and $g_Y(\cdot)$ respectively. What is the pdf of the random variable $Z = X + Y$.

   Answer:

6) Let $x(t)$ be a real-valued continuous-time periodic signal whose Fourier series is given by

$$x(t) \;\; = \;\; \sum_{k=-\infty}^{\infty} a_k \, e^{j\pi kt/4}$$

   where $j \triangleq \sqrt{-1}$. If $a_2 = 1$ then what is the value of a_{-2}.

   Answer:

7) Comment on the linearity and causality of a system whose input signal $x(t)$ and output signal $y(t)$ are related by $y(t) = x(\sqrt{t})$ ($t \geq 0$).

   Answer:

8) Consider a causal LTI system whose input signal is $x(t)$ and output is $y(t)$. The transfer function of the system (i.e., Fourier transform of its impulse response) is given by $H(f) = \frac{3}{4+j6\pi f}$.

   Express $y(t)$ in terms of $x(t)$.

   Answer:

9) Let $x(t) = \sin(2000\pi t)$ be the input signal to a LTI system having impulse response $h(t)$. Under steady state conditions the output signal is given by $y(t) = A\sin(2000\pi t + \phi)$ where $A > 0$.

   Derive an expression for A as a function of $h(t)$.

Answer:

10) If a transmission line of characteristic impedance 50 Ohm has a load of 30 Ohm, what is the reflection coefficient and VSWR?

Answer:

11) Let the electromagnetic field of a plane wave be $\vec{E} = E_0 \sin(Kx + \omega t)\widehat{j}$, where $\omega = 2 \times \pi \times 10^{12}$ rad/s and $K = 2 \times \pi \times 10^4$ rad/m.

What is the speed of the EM wave?

Is this a forward or backward travelling wave?

Answer:

12) Consider a amplitude modulated signal (AM) given by $V(t) = A(1+km(t))\cos(2\pi f_c t)$ where $m(t)$ is the message signal bandlimited to W Hz, and f_c is the carrier frequency. What is the bandwidth of $V(t)$?

Answer:

13) Consider the Frequency Modulated (FM) signal $V(t) = A\cos(2\pi f_c t + 2\pi k \int_0^t m(t)dt\,)$, where $m(t)$ is the bandlimited baseband message signal and f_c is the carrier frequency.
What is the instantaneous frequency of the FM signal $V(t)$ at time t?

Answer: